CHAPTER 3

CHAPTER III - TIME AND MONEY

INTRODUCTION

Modern American agriculture is a complex business. As farms get bigger and investments higher, more and more knowledge is required for determining costs and returns and analyzing alternatives. An understanding and proper use of interest and annuities is necessary in analyzing and comparing the many investments and alternatives available.

Money can be used either to satisfy immediate wants or be invested in capital goods with present or future productive capacity. Rates of interest, or payment for the use of money, are determined by demand, time, and risk. If funds are borrowed, the rate must be at least equal to the rate applicable to the type and length of time the loan is needed. If funds are not borrowed, the rate used will depend on the desire for and opportunity of obtaining returns from using the funds in other productive uses (opportunity costs).

The intent of this chapter is to provide a basic understanding of interest and annuities and how they can be used to compare and analyze investments and alternatives. Most of the interest and annuity factors needed for these calculations can be found in the tables in Chapter 4. Contact your state resource conservationist or economist for tables of other interest rates you may need. This chapter also gives formulas and examples for calculating the factors you may need. To help put things in proper perspective, it is sometimes helpful to draw a sketch or diagram of the situation being analyzed.

TIME VALUE OF MONEY AND OPPORTUNITY COST

Money can be invested and used to make more money over time. Thus, the dollar received today could be put in a bank or invested elsewhere and be worth more than one dollar a year from now. This concept, called the time value of money, is dealt with everyday in home and business finance. For example: landusers may make decisions about purchasing one piece of equipment versus another, or no purchase at all, based on the use of money over time.

The time value of money can be thought of in two forms. First, if the landuser borrows money for a purchase, the time value of money is the interest paid on the loan. If the landuser uses his own money for a purchase, the time value of money would be the return he gave up from another investment, (savings account, certificates of deposit, IRA, etc.), to make the purchase. In this case he has an "opportunity cost;" that is, the interest he would have gotten from a C.D., for example, is now a lost opportunity because he used that money for a purchase.

When a landuser considers purchasing conservation, the time value of money concept applies. There is a cost above and beyond the purchase of the conservation measure. If the landuser borrows to pay for the measure, that additional cost will be equal to the interest he must pay on the loan. If he uses his own money, the additional cost is equal to the return that money would have earned in some other investment.

ONE-TIME VALUES, ANNUAL FLOWS (ANNUITIES) AND LAGS

The benefits and costs of conservation do not necessarily occur at the same time. Certain costs and benefits may occur at one point in time while others occur over a number of years. Some occur today while others occur in the future.

Those values which occur at one point in time are called one-time values. Installation costs are an example of a value which occurs at one-time. Values which occur over time are called annual flows or annuities. Annuities can be generalized into constant, decreasing, and increasing over time, depending on their characteristics. Many of the benefits from conservation fall into the annuity category.

A one-time value can occur today or at some point in the future. If it occurs at some point in the future it is said to be "lagged" or delayed. The replacement cost of a practice is a good example of a lagged one-time value. Annuities too can be lagged. If benefits from a terrace do not start until a year after installation, then those benefits are said to be lagged one year. Deferred grazing following range seeding is another common occurrence of a lagged annuity.

Table 1 illustrates examples of one-time values, annual flows, and lags.

Table 1

One-Time Value	Annual Flow (Ave. Annual Values)	Lagged Values
Installation Cost	Conservation Benefits	Replacement Cost
Replacement Cost	Average Returns	Any value not starting this year
	Average Costs	
	O&M Costs	

AVERAGE ANNUAL VALUES

In order to compare benefits and costs, they must be considered in the same time frame; otherwise we are comparing apples and oranges. A standard form has been developed called average annual values. This term describes an annual flow which is not lagged. In Table 1, the middle column gives 4 examples.

The significance of average annual values is that most businesses, including farming, have accounting systems which are based on average annual values. Therefore, the costs and benefits of conservation, once converted to average annual values, can be added to the costs and returns of the farm business.

Two useful tools for converting benefits and costs of conservation into average annual values are:

1. Amortization key (discussed later in this Chapter)
2. Interest and annuity (I&A) Tables (Chapter 4)

The conversion of costs and benefits of conservation to average annual values without the help of I&A tables would involve the use of many difficult formulas and calculations. The tables were constructed to simplify the process by presenting coefficients developed from the formulas, thus providing much simpler calculations. Formulas and examples are provided for those who would like to use them.

Interest and annuity tables are available for a wide range of interest rates. An interest rate of 10 percent has been used in the following examples. The typical table that SCS uses has seven columns: (1) number of years hence, (2) present value of 1, (3) amortization, (4) present value of an annuity of 1 per year, (5) amount of an annuity of 1 per year, (6) present value of an increasing annuity, and (7) present value of a decreasing annuity. All of these items except "number of years hence" is discussed in detail a little later.

"Number of years hence" is the number of years in which calculations are considered. Several factors may influence this determination: (1) the period may last a year or indefinitely (perpetuity), (2) the measures may have a short or long useful life, or (3) an individual may want to recover his costs in a certain time period.

Three items which are discussed in detail but are not found directly in the tables are: (1) simple interest, (2) compound interest, and (3) sinking fund. These will be illustrated and procedures shown to arrive at the correct factor.

SIMPLE AND COMPOUND INTEREST

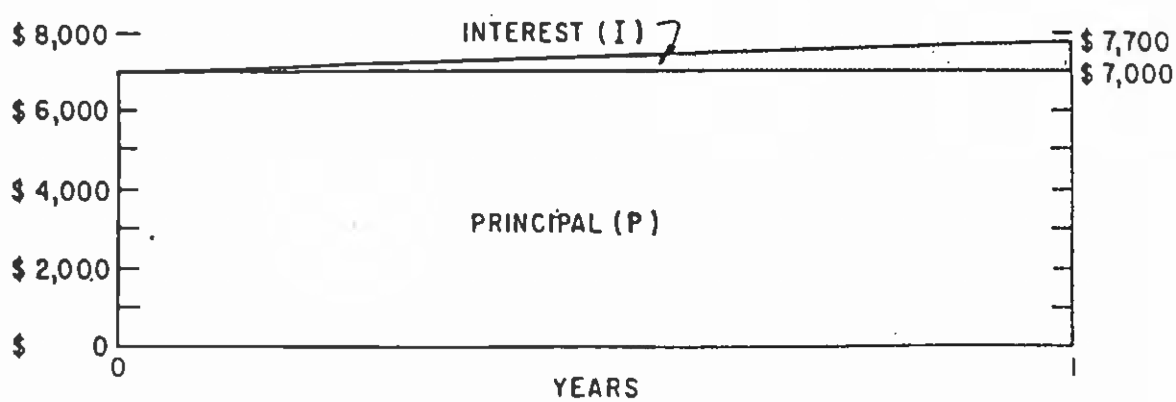
Interest is the earning power of money, what someone will pay you for the use of your money or the rent you pay for the use of someone else's money. Interest is usually expressed as an annual percentage rate (APR) and may be either **simple** interest or **compound** interest.

1. **Simple Interest:** Money paid or received for the use of money, generally calculated over a base period of 1 year at a set interest rate.

Formula: i = (p)(r)(n), where i = interest, p = principal, r = interest rate and n = number of periods (years).

Example: \$7,000 is borrowed at 10 percent interest (APR) for 1 year. How much money will be needed to pay off this loan when it is due?

i = 7,000 x .10 x 1 = \$700 of interest will be due
7,000 of principal will be due
\$7,700 to pay off loan



Example: $3,000 is put into a savings account for 6 months at 10 percent interest (APR), how much interest will be earned?

i = 3,000 x .10 x .5 = $150 will be earned.

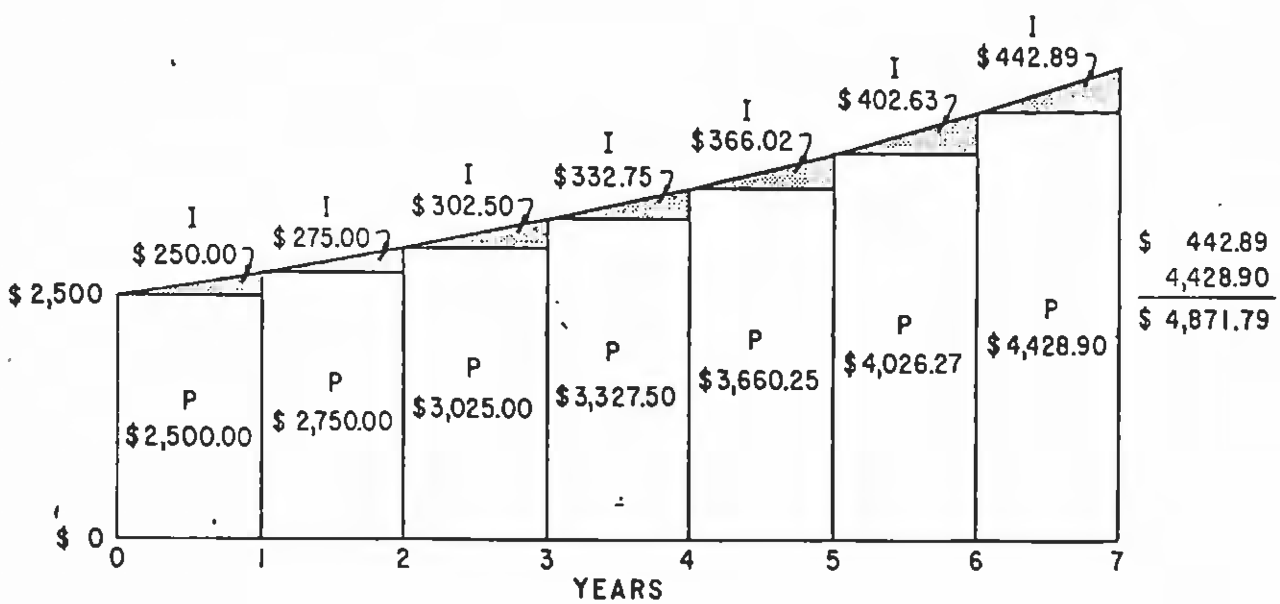
2. **Compound Interest:** Interest that is earned for one period and immediately added to the principle, thus, resulting in a larger principle on which interest is computed for the following period.

Formula: $(1 + i)^n$, where n = number of periods, i = periodic rate of interest, and 1 represents one dollar since the formula results in a factor that is multiplied by the principle dollar amount.

If the interest rate is 10 percent (APR) compounded quarterly for 5 years, then i = .10/4 (4 payments in a year) or .025; n = 5 x 4 (4 payments in a year) or 20. The factor to be multiplied by the principal amount is $(1 + .025)^{20} = 1.63862$.

Example: $2,500 is put into a savings account paying 10 percent interest compounded **ANNUALLY.** How much will be in the account of this depositor at the end of 7 years?

$(1 + .10)^7 = 1.9487171$; 1.9487171 x $2,500 = $4,871.79



If compounded **SEMIANNUALLY** $(1 + .05)^{14} = 1.9799316$;
$1.9799316 \times \$2,500 = \$4,949.83$

If compounded **QUARTERLY** $(1 + .025)^{28} = 1.9964950$;
$1.9964950 \times \$2,500 = \$4,991.24$

If compounded **MONTHLY** $(1 + .0083333)^{84} = 2.0079146$;
$2.0079146 \times \$2,500 = \$5,019.79$
If compounded **DAILY** $(1 + .0002740)^{2555} = 2.0136997$;
$2.0136997 \times \$2,500 = \$5,034.25$

For comparative purposes, compounding gives these results:

$2,500 invested for 7 years at 10 percent---

Compounded annually	$4,871.79
Compounded semiannually	$4,949.83
Compounded quarterly	$4,991.24
Compounded monthly	$5,019.79
Compounded daily	$5,034.25

NOTE: Compound interest factors are not shown by column heading in the tables. However, the same answer can be obtained by dividing by the appropriate "present value of 1" factor since the present value of 1 factor is the reciprocal of the compound interest factor. Since these are "annual" tables, this method will work only if compounding on an annual basis.

Example: Using the preceding problem, what will $2,500 grow to in 7 years at 10 percent interest compounded annually. 1/.51316 (from the interest tables, present value of 1, 7 years hence at 10 percent) = 1.948710 (the same factor was obtained by using the formula). 1.948710 x $2,500 = $4,871.78.

PRESENT VALUE OF 1

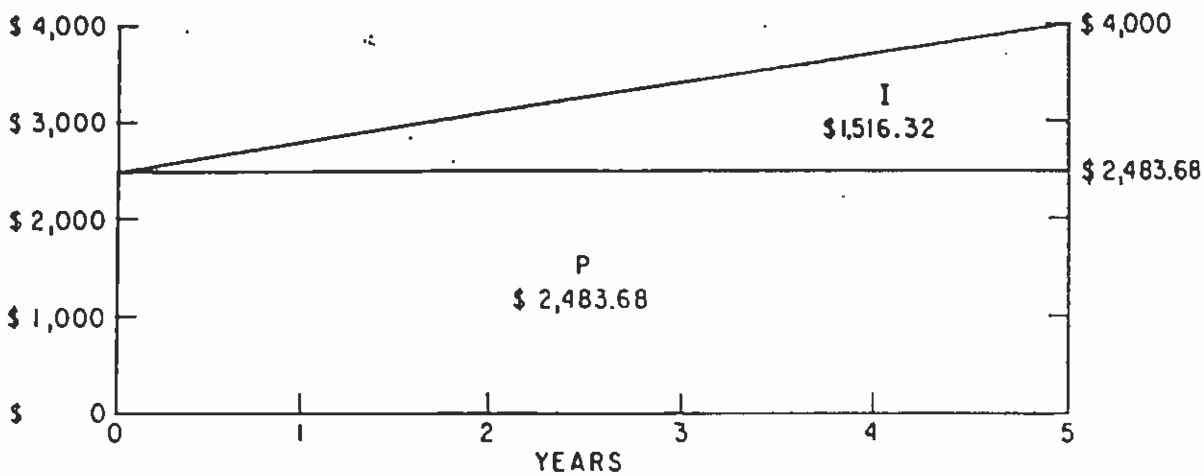
The present value of 1 is the amount that must be invested now at compound interest to have a value of 1 in a given length of time, or what $1.00 due in the future is worth today. It is also known as the present worth of 1 or discount factor.

Formula: $\frac{1}{(1 + i)^n}$ (The "present value of 1" factor is the reciprocal of the "compound interest" factor.)

Example: $4,000 will be needed 5 years from now. How much would need to be invested today at 10 percent interest compounded annually to reach that goal?

$$\frac{1}{(1 + .10)^5} = \frac{1}{1.61051} = .62092$$

.62092 x $4,000 = $2,483.68: this amount would need to be invested now at 10 percent interest compounded annually to be worth $4,000 in 5 years.



The factor can also be found in the 10 percent interest table in the "present value of 1" column for 5 years hence.

Example: What is the discounted value of $10,000 at 10 percent interest for 25 years?

.09230 (from the table) x $10,000 = $923. Looking at it another way, if you invested $923 at 10 percent interest compounded annually and left it alone for 25 years, it will have a value of $10,000 at the end of the 25 years (the power of compounding) or $10,000 to be received in 25 years is worth $923 today.

AMORTIZATION

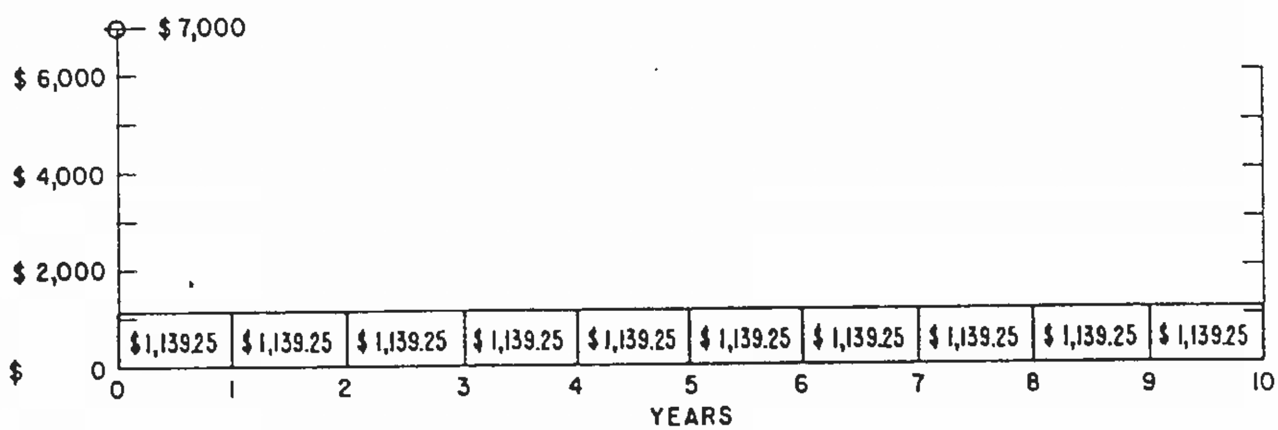
Amortization is also called partial payment or capital recovery factor. It is the extinguishing of a financial obligation in equal installments over time. The amortization factor will determine what annual payment must be made to pay off the principle and interest over a given number of years (average annual cost).

Formula: $\dfrac{i(1+i)^n}{(1+i)^n-1}$ or $\dfrac{i}{1-\dfrac{1}{(1+i)^n}}$

Example: A farmer borrows $7,000 to install a resource management system. The interest rate is 10 percent and the repayment schedule is set up for 10 years. What is his average annual cost, the amount he must pay each year for 10 years to pay off the loan and interest?

$$\frac{.10}{1-\dfrac{1}{(1+.10)^{10}}} = \frac{.10}{1-.38554} = \frac{.10}{.61446} = .16275$$

.16275 x $7,000 = $1,139.25; this amount must be paid each year for 10 years to pay off the $7,000 loan and interest. A total of $11,392.50 will have been paid to close out this loan ($7,000 of principal and $4,392.50 of interest).



The following table displays what occurs each year during the 10 year period.

YEAR	AMOUNT OF LOAN	ANNUAL PAYMENT	PAYMENT PRINCIPAL	PAYMENT INTEREST	REMAINING BALANCE
1	$7000.00	$1139.25	$ 439.25	$ 700.00	$6560.75
2	6560.75	1139.25	483.17	656.08	6077.58
3	6077.58	1139.25	531.49	607.76	5546.09
4	5546.09	1139.25	584.64	554.61	4961.45
5	4961.45	1139.25	643.11	496.14	4318.34
6	4318.34	1139.25	707.42	431.83	3610.92
7	3610.92	1139.25	778.16	361.09	2832.76
8	2832.76	1139.25	855.97	283.28	1976.79
9	1976.79	1139.25	941.57	197.68	1035.22
10	1035.22	1139.25	1035.73	103.52	0
TOTAL	-	$11392.50	$7000.00	$4392.50	-

The factor can also be found in the 10 percent interest table in the "amortization" column for 10 years hence.

NOTE: The amortization factor is the reciprocal of the "present value of an annuity of 1 per year" factor, which means that the same answer can be obtained by dividing by the "present value of an annuity of 1 per year" factor. Using the above problem, the solution is as follows:

$7,000/6.14457 = $1,139.22

AMORTIZATION KEY

In many plant science or botany courses a tool called a "Key" is used to identify plant species by answering a series of questions. This "keying out" process is useful because it allows non-experts to identify species of plants which are unknown to them.
By answering a series of questions, the amortization key serves as a guide for using the interest and annuity tables to convert benefits and costs of conservation to average annual values. The first question on the key is whether the value is an annuity, like benefits from a terrace which flow over time, or if it is a one time value like terrace installation costs.

If it happens to be a one time value, move down the key to the question, "Is it lagged?" A value that will be realized sometime in the future is considered lagged because there is a lag period between now and the time the value takes place. Assuming the value is not lagged, then the only adjustment needed is to amortize the value over the life of the project or evaluation period.

This is accomplished by multiplying the amortization factor found in the tables, times the one time value. This results in an average annual value. Had it been lagged, the one time value would first have to be multiplied by the "present value of one" factor for the lag period, then multiplied by the amortization factor to convert to average annual.

To convert an annuity to an average annual value, it is important to decide if the annuity is constant, increasing or decreasing. If the annuity is a constant flow of value, then it should be multiplied by the "present value of a constant annuity" factor for the period (years) of the annuity. This factor is found in the I&A tables under the column called "present value of an annuity of one per year."

The result of this multiplication would then be multiplied by the amortization factor if the annuity was not lagged. If the annuity period was lagged, it would be multiplied by the "present value of one" factor for the lag period prior to being amortized.

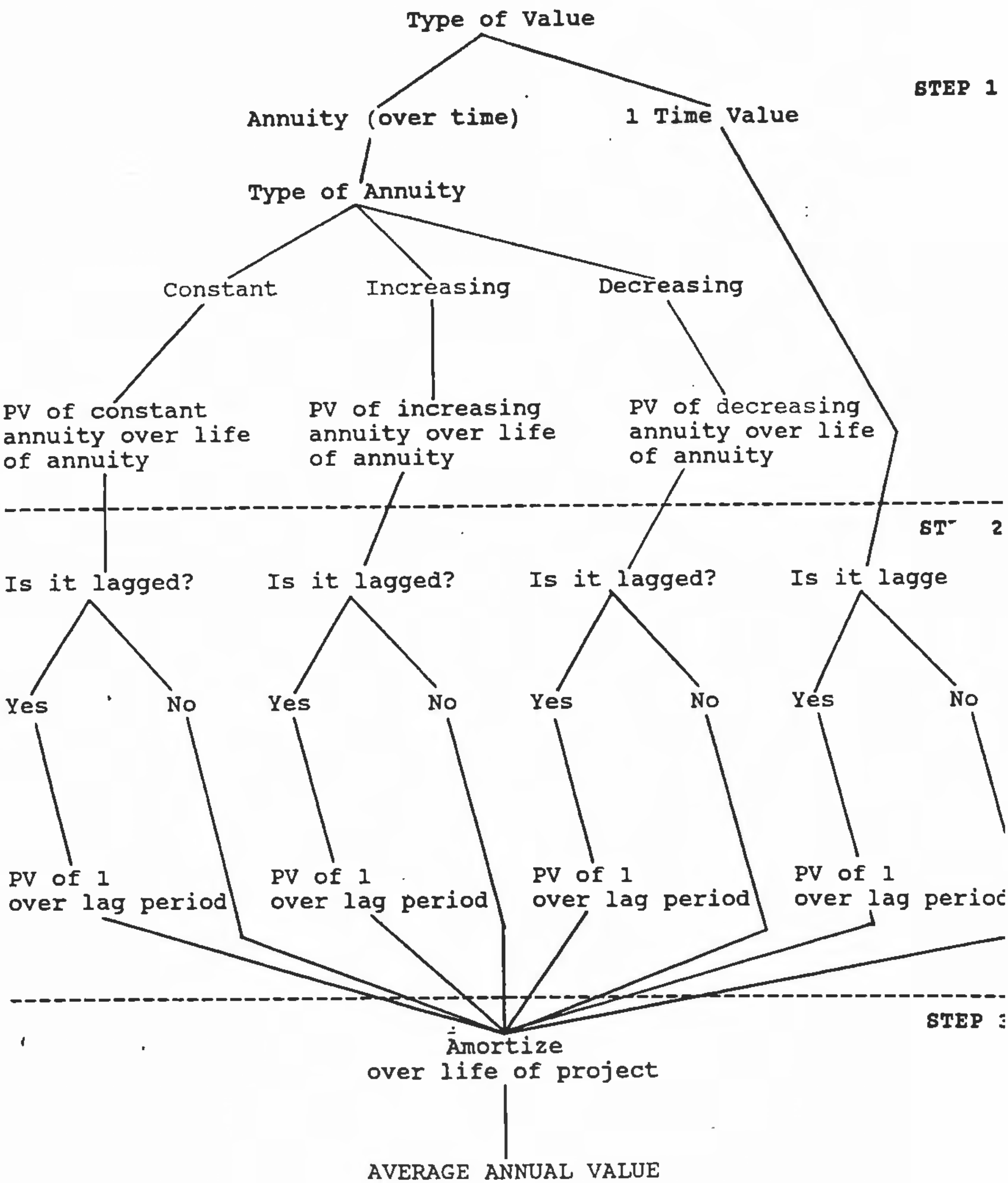
For an increasing or decreasing annuity, recall that the value used to multiply all the factors by, is the yearly average increase or decrease. For example; for an increasing annuity that begins at zero and rises to $500 after 5 years, the yearly average increase would be 500 divided by 5, or 100. That value would be taken times the "present value of an increasing annuity" factor 5 years, which means you locate the factor in the 5 year row under the present value of an increasing annuity column and take it times 100.
If the annuity is lagged, that answer is multiplied by the "present value of one" factor over the lag period, or just amortized if the annuity begins in the first year. The same steps would be taken for a decreasing annuity using the appropriate factors.

To summarize, the first step in the process is to convert any annuity into a one time value. Then we adjust for any lags which are present. And finally, we amortize. Thus we have three basic steps in our process:

1. Convert annuities to one time values
2. Adjust for lags
3. Amortize

Note: Not all steps are used each time. The key guides you through the proper process. For example: if a one time value is considered, the key moves you past step 1. If the annuity or one time value is not lagged, the key moves you past step 2. Remember, this process is necessary to convert benefits and costs of conservation into values which can easily be incorporated into a farmer's records and decisionmaking system.

AMORTIZATION KEY



PRESENT VALUE OF AN ANNUITY OF 1 PER YEAR

Present value of an annuity of 1 per year is also referred to as a constant annuity, present worth of an annuity or capitalization factor.

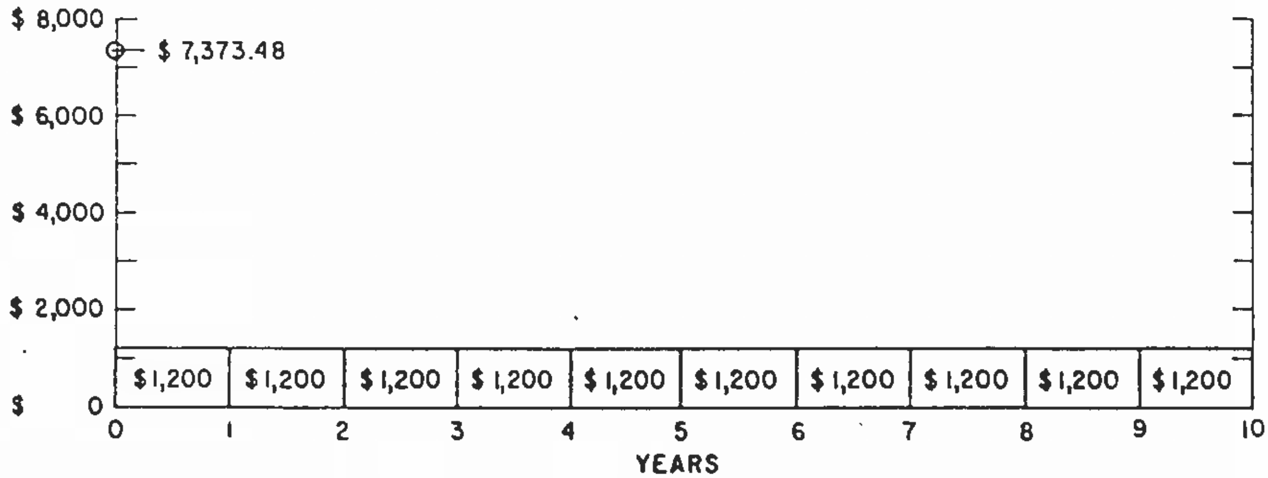
This factor represents the present value or worth of a series of equal payments or deposits over a period of time. It tells us what an annual deposit of $1.00 is worth today. If a fixed sum is to be deposited or earned annually for "n" years, this factor will determine the present worth of those deposits or earnings.

Formula: $\dfrac{(1 + i)^n - 1}{i(1 + i)^n}$

Example: You want to provide someone with $1,200 a year for 10 years. The interest rate is 10 percent. How much do you need to deposit to produce $1,200 a year for 10 years?

$$\frac{(1 + .10)^{10} - 1}{.10(1 + .10)^{10}} = \frac{(1.10)^{10} - 1}{.10(2.59374)} = \frac{1.59374}{.259374} = 6.14457$$

6.14457 x $1,200 = $7,373.48; this amount must be deposited now to produce an annuity of $1,200 for 10 years. A total of $12,000 will have been received from this one-time deposit of $7,373.48. The interest amounts to $4,626.52.



The factor can also be found in the 10 percent interest table in the "present value of an annuity of 1 per year" column for 10 years hence.

NOTE: The factor is the reciprocal of the "amortization" factor. Therefore, the same answer can be obtained by dividing by the amortization factor.

$1,200/.16275 = $7,373.27

AMOUNT OF AN ANNUITY OF 1 PER YEAR

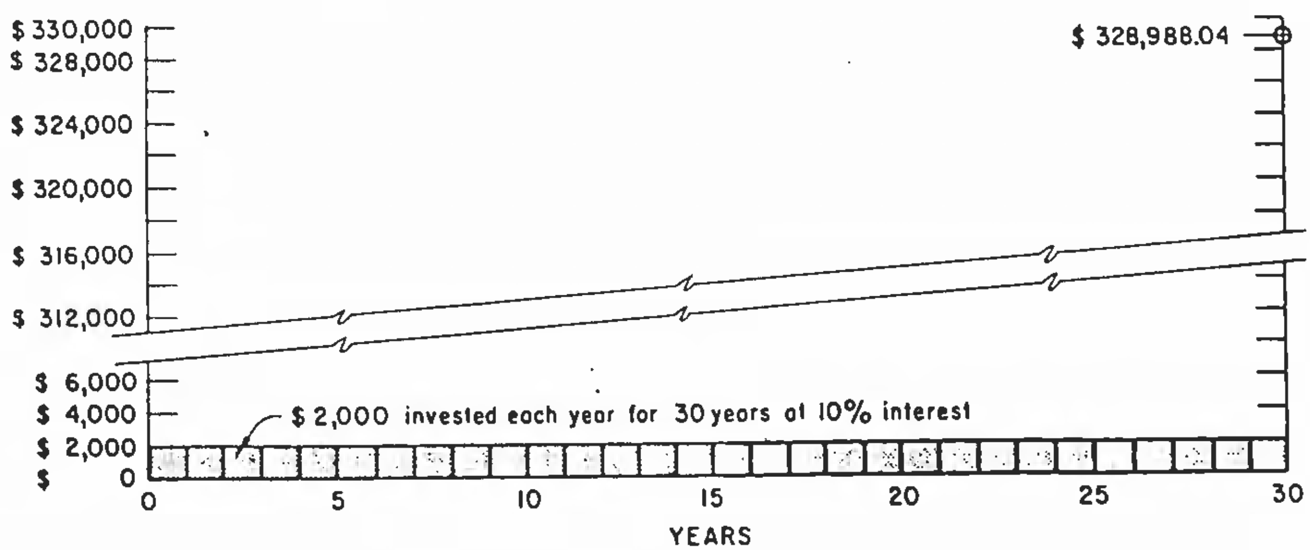
The amount of an annuity of 1 per year is the amount that an investment of $1 per year will accumulate in a certain period of time at compound interest.

Formula: $\frac{(1 + i)^n - 1}{i}$

Example: $2,000 per year will be invested in an individual retirement account (IRA) for 30 years paying 10 percent interest compounded annually. What will be the value of this account at the end of the 30 years?

$$\frac{(1 + .10)^{30} - 1}{.10} = \frac{16.449402}{.10} = 164.49402$$

164.49402 x $2,000 = $328,988.04; value of the IRA account at the end of 30 years.



The factor can also be found in the 10 percent interest table in the "amount of an annuity of 1 per year" column for 30 years hence.

SINKING FUND

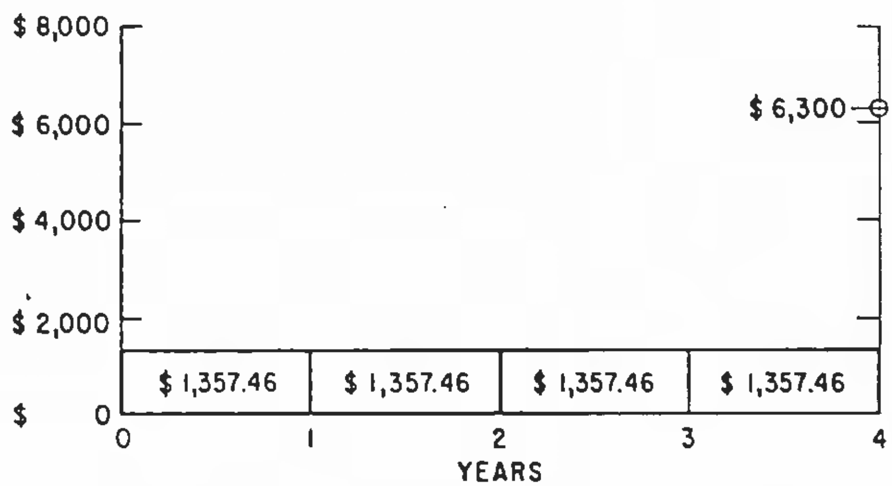
The sinking fund factor is used to determine what size annual deposit will be required to accumulate a certain amount of money in a certain number of years at compound interest.

Formula: $\dfrac{i}{(1+i)^n - 1}$

Example: $6,300 will be needed in 4 years. What amount will need to be deposited each year at 10 percent compound interest to reach this goal?

$$\frac{.10}{(1+.10)^4-1} = \frac{.10}{.4641} = .21547$$

.21547 x $6,300 = $1,357.46; this amount must be deposited annually for 4 years at 10 percent interest, compounded annually to accumulate the $6,300.



NOTE: The sinking fund factor is not shown in the tables but the same answer can be obtained by dividing by the appropriate "amount of an annuity of 1 per year" factor, this is because the amount of an annuity of 1 per year factor is the reciprocal of the sinking fund factor.

$6,300/4.64100 = $1,357.46

NOTE: The sinking fund factor is also equal to the amortization factor minus the interest rate.

.31547 - .10 = .21547; .21547 x $6,300 = $1,357.46

PRESENT VALUE OF AN INCREASING ANNUITY

The present value of an increasing annuity is a measure of present value of an annuity that is not constant but increases uniformly over a period of time. When using this factor, it is important to note that the value of $1 (which is multiplied by the factor) is the annual rate of increase and not the total increase during the period.

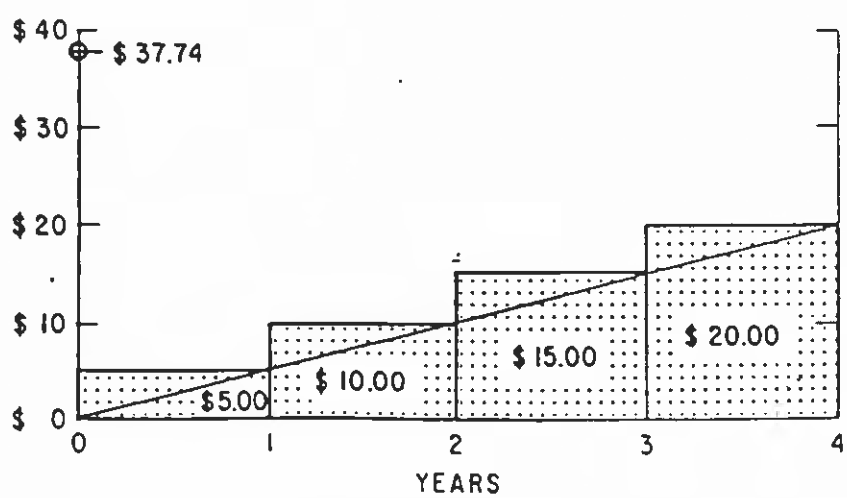
Formula: $\frac{(1 + i)^{n+1} - (1 + i) - n(i)}{(1 + i)^{n}(i)^{2}}$

Example: A farmer renovates a pasture and estimates that it will reach full production in 4 years. The improvement will increase uniformly over the 4-year period and at full production will improve net income $20 per year per acre. Using an interest rate of 10 percent, what is the present value of this increasing annuity?

$$\frac{(1 + .10)^{5} - (1 + .10) - 4(.10)}{(1 + .10)^{4}(.10)^{2}} = \frac{1.61051 - 1.1 - .4}{1.46410 \times .01} = \frac{.11051}{.014641} =$$

7.54798

We now need to determine the annual rate of increase. The annual rate of increase is $20/4 or $5. This is to say that the annuity is not constant or the same each year but that he will receive income of $5 the first year, $10 the second year, $15 the third, and $20 the fourth year (increases uniformly at $5 per year). The present value of this increasing annuity or income stream is then 7.54798 x $5 or $37.74. This also means that if you deposited $37.74 in an account paying 10 percent interest compounded annually, you could withdraw $5 at the end of year 1, $10 at the end of year 2, $15 at the end of year 3, and $20 at the end of year 4, and there would then be a balance of $0.00.



The factor can also be found in the 10 percent interest tables in the "present value of an increasing annuity" column for 4 years hence.

PRESENT VALUE OF A DECREASING ANNUITY

The present value of a decreasing annuity factor is used to determine how much something is presently worth that will provide an annuity that decreases uniformly each year. Again, it is important to note that the value of $1 (which is multiplied by the factor) is the annual rate of decrease and not the total decrease during the period.

Formula: $$\frac{n(i)-1+\frac{1}{(1+i)^n}}{(i)^2}$$

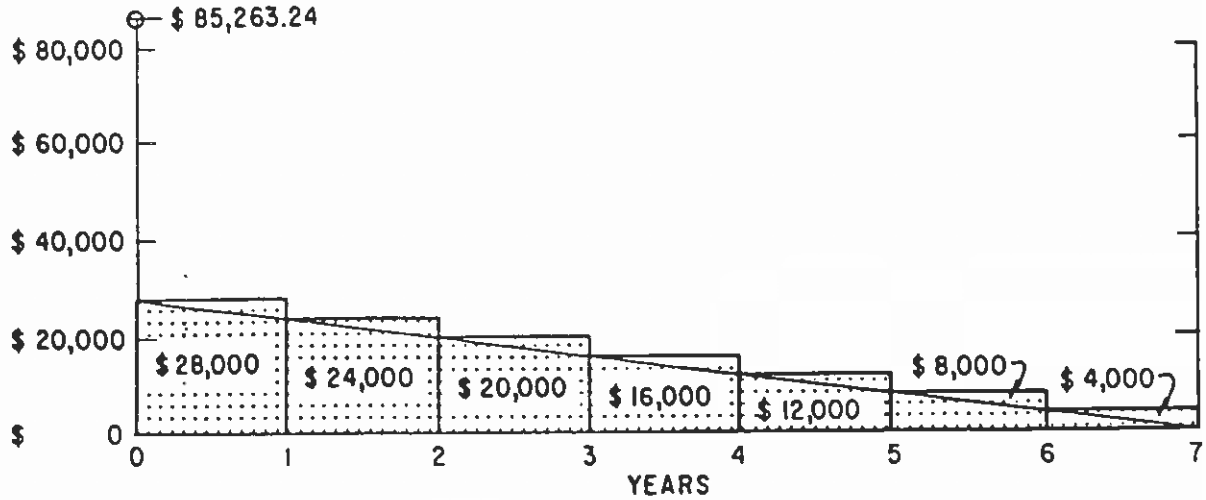
Example: A gravel pit is producing $28,000 income annually. Due to a decreasing supply which is costlier to remove, income will drop at a steady rate until it equals zero in 7 years. At 10 percent interest, what is the present value of the gravel?

$$\frac{7(.10)-1+\frac{1}{(1+.10)^7}}{(10)^2} = \frac{-.3+\frac{1}{1.1^7}}{.01} = \frac{-.3+.513158}{.01} =$$

$$\frac{.213158}{.01} = 21.31581$$

We now need to determine the annual rate of decrease. The annual rate of decrease is $28,000/7 or $4,000.00. This is to say that the annuity is not constant or the same each year, but that he will receive income of $28,000 the first year, $24,000 the second, $20,000 the third, etc., until the supply runs out on the seventh year and becomes $0.00:

The present value of this decreasing annuity or income stream is then 21.31581 x $4,000 or $85,263.24; this is the amount that would need to be deposited now to produce the identified decreasing annuity.

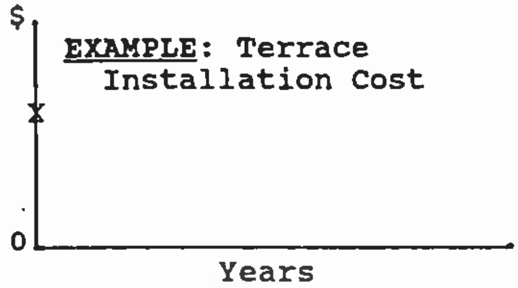


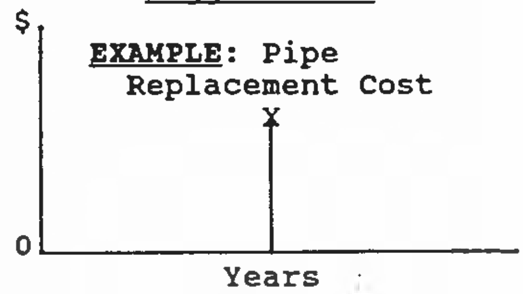
The factor can also be found in the 10 percent interest table in the "present value of a decreasing annuity" column for 7 years hence.

VALUE DIAGRAMS

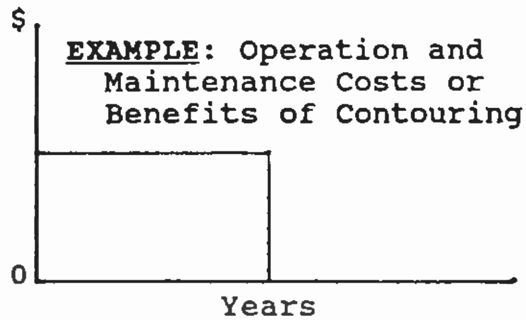
Type Of Value	Non-Lagged Value	Lagged Value

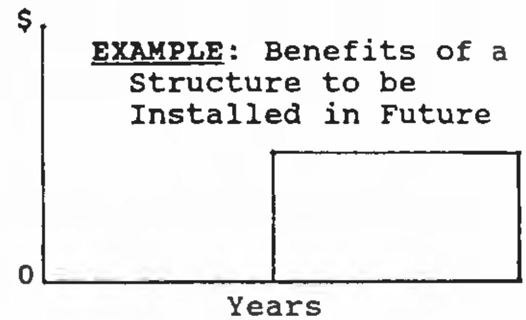
One-Time Value



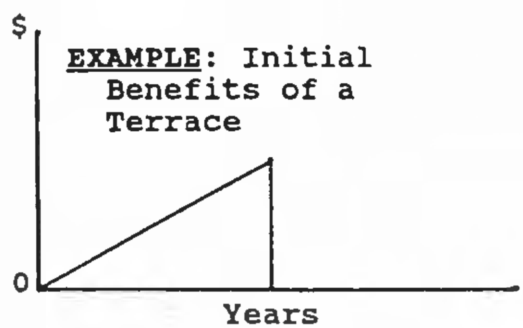


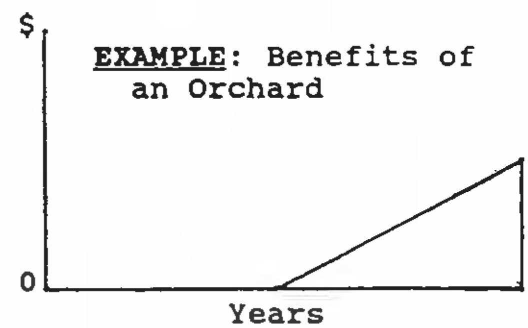
Constant Annuity



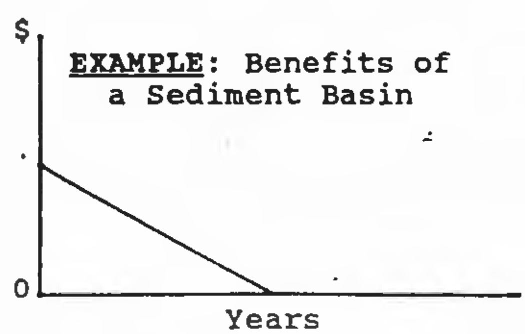


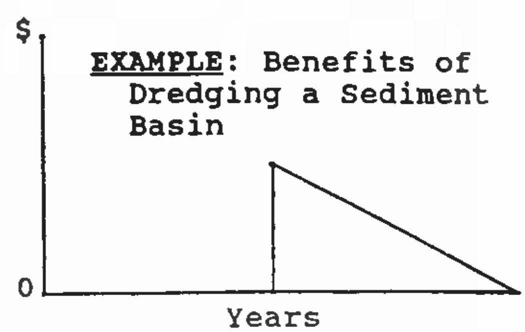
Increasing Annuity





Decreasing Annuity





RULE OF 72

The rule of 72 states that 72 divided by the interest rate received will result in the number of years it will take to double your money at compound interest.

<u>**EXAMPLE**</u>: At 8 percent compound interest how long will it take to double an investment of $150?

$$\frac{72}{8} = 9 \text{ years to double your money (\$300)}$$

PROOF: PV of 1, 9 years hence, at 8% = .50025 (from I&A Tables). .50025 x $300 = $150

OR

Dividing 72 by the number of years you want to double your money in, will result in the interest rate needed to double the investment.

<u>**EXAMPLE**</u>: At compound interest, what interest rate would you need to receive in order to double $150 in 9 years?

$$\frac{72}{9} = 8 \text{ percent return needed to double your investment in 9 years.}$$

PROOF: $150/.50025 = $300

> NOTE: Compound interest factors are not shown by column heading in the I&A tables. However, the answer can be obtained by dividing by the appropriate "present value of 1" factor (.50025) since the present value of 1 factor is the reciprocal of the compound interest factor. Since these are "annual" tables, this method will work only if compounding on an annual basis.