CHAPTER 4

CHAPTER IV - COMPOUND INTEREST AND ANNUITY TABLES

INTRODUCTION

This chapter was developed by the West National Technical Center (WNTC) at Portland, Oregon. Much of this material has been covered in Chapter III, however the section titled, **CLUES TO PROBLEM SOLVING BY THE USE OF DIAGRAMS & SKETCHES** starting on page 5 and the section, **EXAMPLE PROBLEMS** starting on page 9 are especially helpful in understanding and solving a variety of economic evaluations.

COMPOUND INTEREST AND ANNUITY TABLES

These interest tables are provided as a reference to enable the user to properly account for the effects of interest and time in making an economic analysis. The basic principles of the time value of money, and the use of interest factors in making comparisons between values that occur at different points in time, are presented.

The tables have four elements in common: (a) an amount, (B) an interest rate, (C) a term, and (D) a payment. If any three of these elements are known, then the fourth can be derived from the tables.

Procedures for discounting future benefits and costs or otherwise converting benefits and costs to a common time basis are also presented.

BASIC DEFINITIONS

Value — In economics, value represents any quantity expressing the worth of something. In resource development projects, value is used to express benefits arising from effects of project measures, or it could be the cost for providing such measures.

Number of Years Hence — This is the number of periods (years, months, or days) in which calculations are considered. There may be many conditions which influence this determination: (1) a benefit may last a year or indefinitely (perpetuity), (2) the measures may have a short or long useful life, (3) the period of evaluation may be set by policy, (4) an individual may want to recover his costs in a certain time period, or (5) costs or returns may occur over varying time periods or at varying rates for the same period.

Annuity — Annuity is a series of equal payments made at equal intervals of time. The most common type of annuity is our paychecks, at least those that meet the equal payment requirement. Annuity may be a benefit (to those receiving equal sums of money) or a cost (to those making the payment).

Interest — Interest is economic rent of money. When money is borrowed, the amount borrowed must be repaid along with a use charge called interest. Or, said another way, interest is money paid for the use of money.

The appropriate rate of interest will depend upon the situation or the reason for the analysis. Demand, time, and risk (includes inflation) determine the rate of interest charged or paid in commercial lending establishments. If personal money is used in lieu of borrowed or lent funds, an opportunity cost[1/] should be taken into consideration.

Interest rate is expressed as a percent of the principal amount and is understood to be an interest rate per year. There are two kinds of interest--simple and compound.

[1/] Opportunity cost is the return forgone from the most likely alternative use of money.

Simple Interest

Simple interest is rent on the principle amount only. If $100 is loaned or borrowed and a year later $108 is repaid, the $8 is interest and, since it was for one year, the interest rate is 8 percent. The amount of interest is computed by the following formula:

$i = (p)(r)(n)$, where i = interest, p = principle ($100), r = periodic interest rate (8%), and n = number of periods (1 year).

$i = (\$100)(.08)(1) = \$8.00$

Compound Interest

Compound interest is interest that is earned for one period and immediately added to the principle, yielding a larger principle on which interest is computed for the following period. This means the accrued unpaid interest is actually converted to additional principle.

Problem: What will $500 grow to in 5 years at 8 percent interest?

Solution: ($500)(1.46933 1/) = $734.67

The compound interest factor is represented by the following formula:

$(1 + i)^n$, where n is the number of periods, i is the periodic rate of interest, and 1 represents one dollar since the formula results in a factor that is multiplied by the principle dollar amount.

NOTE: Compound interest factors are not shown by column heading in the tables, but the same answer can be obtained by dividing the appropriate "present value of 1" factor since the present value of 1 factor is the reciprocal of the compound interest factor. Using the preceding problem: What will $500 grow to in 5 years at 8 percent interest? The solution is as follows:

Solution: $500 ÷ .68058 2/ = $734.67

Present Value of 1

Sometimes called present worth of 1 is what $1.00 due in th future is worth today or at present. The present value of a specified single sum of money due at some named future date is that sum of money which, if put at compount interest for the same time period would have a compound amount equal to the specified amount. Hence, this is the reason for the factor being occasionally called the "discount factor." Delayed cost or benefits can be reduced to present worth at year 0 with this factor.

Problem: At 8 percent interest find the present value of $1,000 to be received in 5 years.

Solution: ($1,000)(.68058 3/) = $680.58

In other words, $1,000 to be received in 5 years is worth $680.58 today.

The present value of 1 factor is represented by the following formula.

$$\frac{1}{(1 + i)^n}$$

NOTE: The present value of 1 factor is the reciprocal of the "compound interest" factor.

1/Compound interest, 5 years hence, 8 percent interest (not shown in tables).
2/Present value of 1, 5 years hence, 8 percent interest.
3/Present value of 1, 5 years hence, 8 percent interest.

Amortization

Amortization is sometimes called partial payment or capital recovery factor. This factor will convert capital or initial cost to annual cost. It will determine what annual payment including interest must be made to pay off the initial cost over a given number of years.

Problem: At 8 percent interest, find the annual equivalent investment cost over a period of 10 years of a facility with an initial cost of $1,000.

Solution: Annual cost = initial cost multiplied by the amortization factor for 10 years at 8 percent interest.

Annual cost = ($1,000)(.14903 1/)

Annual cost = $149.03

The formula for the amortization factor is expressed as:

$$\text{Amortization} = \frac{i\,(1+i)^n}{(1+i)^n - 1} \text{ or } \frac{i}{1 - \frac{1}{(1+i)^n}}$$

NOTE: The amortization factor is the reciprocal of the "present value of an annuity of 1 per year" factor which means that the same answer can be obtained by dividing by the present value of an annuity of 1 per year factor. For example, using the above problem the solution is as follows:

$1,000 ÷ 6.71008 2/ = $149.03

Present Value of an Annuity of 1 Per Year

Present value of an annuity of 1 per year also referred to as constant annuity, present worth of an annuity, or capitalization factors.

This factor represents the present value or worth of a series of equal deposits over a period of time. It tells us what an annual deposit of $1.00 is worth today. If a fixed sum is to be deposited or earned annually for "n" years, this factor will determine the present worth of those deposits or earnings.

Problem: If $600 will be placed in your savings account each year for 10 years, what is the present worth of the total amount or if you want to give $600 per year to someone for 10 years what sum would be required at present? The interest rate is 8 percent.

Solution: $600 x 6.71008 2/ = $4,026

This is the present value of receiving $600 per year for 10 years or it is the amount that would need to be deposited today to make annual withdrawals of $600 for 10 years.

The present value of an annuity of 1 per year factor is expressed as follows:

$$\frac{(1+i)^n - 1}{i\,(1+i)^n}$$

NOTES:

a. The factor is the reciprocal of the "amortization" factor. Therefore, the same answer can be obtained by dividing by the amortization factor:

Solution: $600 ÷ .14903 1/ = $4,026

1/Amortization, 10 years hence, 8 percent interest.
2/Present value of an annuity of 1 per year, 10 years hence, 8 percent interest.

b. "Amount of an annuity of 1 per year" multiplied by "present value of 1" = "present value of an annuity of 1 per year."

Amount of an Annuity of 1 Per Year

Amount of an Annuity of 1 per year is also called "accumulation of an annuity." As stated earlier an annuity is a sequence of equal payments made at uniform intervals with each payment earning compound interest during its respective earning term. The amount of an annuity of 1 per year factor shows how much an annuity, invested each year, will grow over a period of years. It can also show how much it is worth to provide protection against losing the opportunity of investing an annuity each year. For example, preventing erosion that causes a loss of net income of $25 per year for 50 years has an accumulated value of $14,344 at the end of 50 years at 8 percent interest (25 x 573.77016)[1/].

The factor for an amount of an annuity of 1 per year is expressed as follows:

$$\frac{(1 + i)^n - 1}{i}$$

Another example is a person establishing a retirement reserve. If $500 is placed in a savings account, each year earning 8 percent, what is the amount of the reserve at the end of 20 years?

Solution: ($500)(45.76196[2/]) = $22,881

Sinking Fund

This factor is used to determine what size annual deposit will be required to accumulate to a certain amount (given) in a certain number (given) of years at compound interest.

Problem: If $25,000 is needed to meet a term note due in 10 years, what amount will need to be deposited each year at 8 percent compound interest to reach the goal?

Solution: ($25,000)(.069029[3/]) = $1,726

The sinking fund factor is expressed as:

$$\text{Sinking Fund} = \frac{i}{(1 + i)^n - 1}$$

NOTE: The sinking fund factor is not shown by column heading in the tables but the same answer can be obtained by <u>dividing</u> by the appropriate "amount of an annuity of 1 per year" factor since the amount of an annuity of 1 per year factor is the reciprocal of the sinking fund factor.

Solution: $25,000 ÷ 14.48656[4/] = $1,726

Present Value of an Increasing Annuity

This factor shows how much something is presently worth that will provide increasing sums of money over a period of years.

It should be noted that to meet the definition of an annuity (equal payments at equal intervals) the increases must be uniform. For example, seeding a field to grass may eventually be worth $18 per acre in increased net income, but it will take 6 years before this value is realized. The annuity is increasing $3 per year (18 ÷ 6) so the definition of an annuity is satisfied. The total annuity we will receive is not level or the same each year since

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1/Amount of an annuity of 1 per year, 50 years hence, 8 percent interest.
2/Amount of an annuity of 1 per year, 20 years, 8 percent interest.
3/Sinking fund factor 10 years hence, 8 percent interest (not shown in tables).
4/Amount of an annuity of 1 per year, 8 percent interest, 10 years hence.

we will receive \$3 the first year, \$6 the second year, and \$9 the third year until at the end of the sixth year the \$18 per year will be realized. At 8 percent interest the present value or present worth of something that will provide this amount of flow or build up of funds if \$45.44 (\$3 x 15.14615[1/]).

The factor for "present value of an increasing annuity" is expressed in the following formula:

$$\frac{(1+i)^{n+1} - (1+i) - n(i)}{(1+i)^n (i)^2}$$

Present Value of a Decreasing Annuity

This factor tells us how much something is presently worth that will provide an annuity that gets less each year.

The decrease must be uniform to meet the definition of an annuity. For example, a sediment pool will return \$1,000 recreation benefits the first year but at the end of 40 years, because of sediment accumulation, it will have no value.

The flow of funds is decreasing uniformly at \$25 per year (\$1,000 ÷ 40 years). Therefore, the basic definition of annuity is met. Benefits would be \$1,000 the first year, \$975 the second year, \$950 the third year, etc., until at the end of 40 years there would be no returns.

The present value of this sediment pool at 8 percent interest is \$8,774 (\$25 x 350.94233)[2/]. The \$8,774 invested at 8 percent interest would return the decreasing annuities described.

The factor for "present value of a decreasing annuity" is displayed in the following formula:

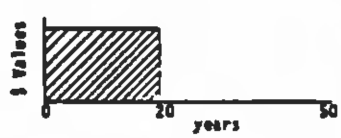
$$\frac{n(i) - 1 + \frac{1}{(1+i)^n}}{(i)^2}$$

Clues to Problem Solving by the use of Diagrams & Sketches

The solution to complicated problems is made easier by the use of a sketch or diagram. There are six basic diagrams that can be combined or used to illustrate all problems involving decreases, increases, delays, or discount by the use of the compound interest and annuity tables. By using the graphic illustrations and lettering the basic parts, all parts of the more complicated problems can be broken down into a series of simple familiar steps, each one easy to understand and solve. You will also save time and make fewer mistakes.

Clue 1

Situation: A return of \$600 will be received each year for 20 years. What is the average annual value for a 50-year evaluation period at 8 percent interest?



Step 1 - Determine present worth of benefits occurring evenly over each of the 20 years. Multiply \$600 by the factor for the "present value of annuity of 1 per year" corresponding to the length of return (20 years).

(\$600)(9.81815)[3/] = \$5,891

1/Present value of an increasing annuity, 6 years hence, 8 percent interest.
2/Present value of decreasing annuity, 40 years, 8 percent interest
3/Present value of an annuity of 1 per year, 20 years, 8 percent interest.

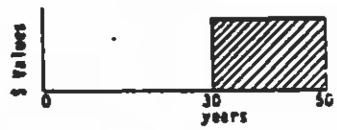
Step 2 - Determine the annual equivalent value of the income stream. Multiply the $5,891 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($5,891)(.08174)1/ = $482

Conclusion: The average annual value is $482.

Clue 2

Situation: A return of $600 will be received for 20 years. However, it will be 30 years before the return will begin. What is the average annual value for a 50-year evaluation period at 8 percent interest?



Step 1 - Determine present worth of benefits occurring evenly over each of the 20 years (50-30). Multiply $600 by the factor for "present value of an annuity of 1 per year" corresponding to the length of return (20 years).

(600)(9.81815 2/) = $5,891

Step 2 - Discount the benefits that are delayed 30 years. Multiply the $5,891 by the discount factor "present value of 1" corresponding to the length of delay (30 years).

($5,891)(.09938)3/ = $585

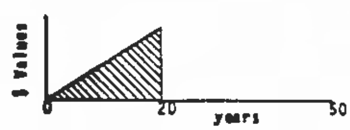
Step 3 - Determine the annual equivalent value of the capital sum. Multiply the $585 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($585)(.08174)1/ = $48

Conclusion: The average annual value is $48.

Clue 3

Situation: A return will begin to build from $0 in year 0 to $600 in 20 years then cease. What is the average annual benefit at 8 percent interest for a 50-year evaluation period?



Step 1 - To meet the definition of an annuity (a series of equal payments made at equal intervals of time, page 1), where values are uniformly increasing, divide the ending value reached by the number of years to reach the value.

$\frac{\$600}{20}$ = $30 per year

Step 2 - Determine present worth of the income stream. Multiply the $30 per year by the "present value of an increasing annuity" corresponding to the number of years the return is received (20 years).

($30)(78.90794)4/ = $2,367

1/Amortization, 50 years, 8 percent interest.
2/Present value of an annuity of 1 per year, 20 years, 8 percent interest.
3/Present value of 1, 30 years, 8 percent interest.
4/Present value of an increasing annuity of 1 per year, 20 years, 8 percent interest.

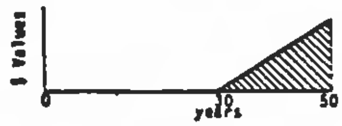
Step 3 - Determine annual equivalent value of the income stream. Multiply the $2,367 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($2,367)(.08174)[1/] = $193

Conclusion: The average annual benefit is $193.

Clue 4

Situation: A return will begin to build from $0 in year 30 to $600 in year 50. What is the average annual benefit at 8 percent interest for a 50-year evaluation period?



Step 1 - To meet the definition of an annuity (a series of equal payments made at equal intervals of time, page 1). Where values are uniformly increasing, divide the ending value reached by the number of years to reach the value.

$\frac{\$600}{20}$ = $30 per year

Step 2 - Determine present worth of the income stream beginning in year 30. Multiply the $30 per year by the "present value of an increasing annuity" corresponding to the number of years the return is received (20 years).

($30)(78.90794)[2/] = $2,367

Step 3 - Discount the income stream that are delayed 30 years. Multiply the $2,367 by the discount factor (present value of 1) corresponding to the length of delay (30 years).

($2,367)(.09938)[3/] = $235

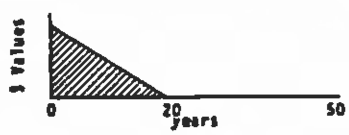
Step 4 - Determine the annual equivalent value of the income stream. Multiply the $235 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($235)(.08174)[1/] = $19

Conclusion: The average annual benefit is $19

Clue 5

Situation: A present return of $600 will uniformly decrease to $0 at year 20. What is the average annual value of 8 percent interest rate for a 50-year evaluation period?



Step 1 - To meet the definition of an annuity (a series of equal payments made at equal intervals of time, page 1), where values are uniformly decreasing, divide the beginning value by the number of years to reach $0.

$\frac{\$600}{20}$ = $30

1/Amortization, 50 years, 8 percent interest.
2/Present value of an increasing annuity, 20 years, 8 percent interest.
3/Present value of 1, 30 years, 8 percent interest.

Step 2 - Determine the present worth of the income stream. Multiply the $30 per year by the "present value of a decreasing annuity" corresponding to the number of years the return is to be received (20 years).

($30)(127.27316)[1/] = $3,818

Step 3 - Determine the annual equivalent value of the income stream. Multiply the $3,818 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($3,818)(.08174)[2/] = $312

<u>Conclusion</u>: The average annual benefit is $312.

Clue 6

Situation: After year 30, return of $600 uniformly decreases to $0 at year 50. What is the average annual value at 8 percent interest rate?

Step 1 - To meet the definition of an annuity (a series of equal payments made at equal intervals of time, page 1) where values are uniformly decreasing, divide the beginning value by the number of years to reach $0.

$\frac{\$600}{20}$ = $30 per year

Step 2 - Determine present worth of the income stream in beginning in year 30. Multiply the $30 per year by the "present value of a decreasing annuity" corresponding to the number of years the return is received (20 years).

($30)(127.27316)[1/] = $3,818

Step 3 - Discount the income stream that's delayed 30 years. Multiply the $3,818 by the discount factor (present value of 1) corresponding to the length of delay (30 years).

($3,818)(.09938)[3/] = $379

Step 4 - Determine the annual equivalent value of the income stream. Multiply the $379 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($379)(.08174)[2/] = $31

<u>Conclusion</u>: The average annual value is $31.

1/Present value of a decreasing annuity, 20 years, 8 percent interest.
2/Amortization, 50 years, 8 percent interest.
3/Present value of 1, 30 years, 8 percent interest.

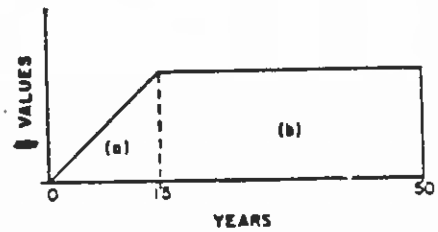
Example Problems

The following problems illustrate the use of the compound interest and annuity tables. The purpose of the problems is to illustrate the principles used in applying each of the annuity factors described in the previous section.

Problem 1

A net return will increase in uniform amounts to $3,000 for a 15-year period. Thereafter, through the remaining 35 years of the evaluation period, the benefits will remain constant at 8 percent interest. Determine the annual equivalent value over the 50-year evaluation period.

First diagram the problem.



Solution:

a. $3,000 ÷ 15 = $200 per year increase per year

($200)(56.44514[1/]) = $11,289

b. ($3,000)(.31524[2/])(11.65457[3/]) = $11,022

Annual Equivalent Value = $11,289 + $11,022 = $22,311
($22,311)(.08174[4/]) = $1,824

Shortcut - Table A (page 14) provides straight line lag discount factors that can be used directly for selected percents of interest.

The illustration of the shortcut method in solving the above problem is as follows:

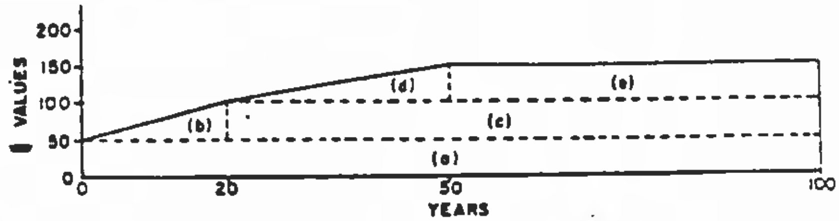
($3,000)(0.608[5/]) = $1,824

1/Present value of an increasing annuity, 15 years hence, 8 percent interest.
2/Present value of 1, 15 years hence, 8 percent interest.
3/Present value of an annuity of 1 per year, 35 years hence, 8 percent interest.
4/Amortization, 50 years, 8 percent interest.
5/Table 1, page 14, 15 year lag, 50 year evaluation period, 8 percent interest.

Problem 2

Land use will become more intensive in a flood prevention-drainage project as the operators accept and utilize the protection made possible by the project. Presently net benefits per acre are $50. They are expected to uniformly increase to $95 per acre in 20 years. Thence, to $149 by the end of 50 years, and then remain uniform for the remainder of the projects life. Determine the average value per acre if the interest rate is 8 percent and the evaluation period is 100 years.

Diagram the problem:



Solution:

a. ($50)(12.49432[1/]) = $625

b. $95 - $50 = $45

45 ÷ 20[2/] = $2.25 per year

($2.25)(78.90794[3/]) = $178

c. ($45)(12.47351[4/])(.21455[5/]) = $120

d. $149 - $95 = $54

54 ÷ 30[6/] = $1.80 per year

($1.80)(114.71358[7/])(.21455[5/]) = $44

e. ($54)(12.23348[8/])(.02132[9/]) = $14

The present value of receiving the above benefits is $981 (625 + 178 + 120 + 44 + 14).

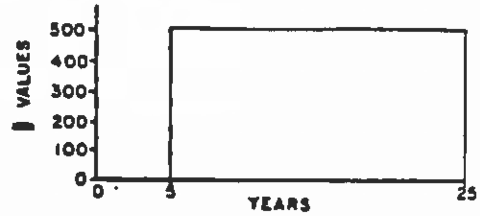
The average annual value per acre is $78.52 (981 x .08004[10/]).

1/Present value of an annuity of 1 per year, 100 years, 8 percent interest.
2/Necessary to meet the definition of an annuity.
3/Present value of an increasing annuity, 20 years, 8 percent interest.
4/Present value of an annuity of 1 per year, 80 years, 8 percent interest.
5/Present value of 1, 20 years, 8 percent interest.
6/Necessary to meet the definition of an annuity, (50 - 20) = 30 years.
7/Present value of an increasing annuity, 30 years, 8 percent interest.
8/Present value of an annuity of 1 per year, 50 years, 8 percent interest.
9/Present value of 1, 50 years, 8 percent interest.
10/Amortization, 100 years, 8 percent interest.

Problem 3

It will take 5 years for seeding on rangeland to become established after which it will yield a value of $500 on a continuing basis. What is the average annual benefit for the 25-year evaluation period at 8 percent interest?

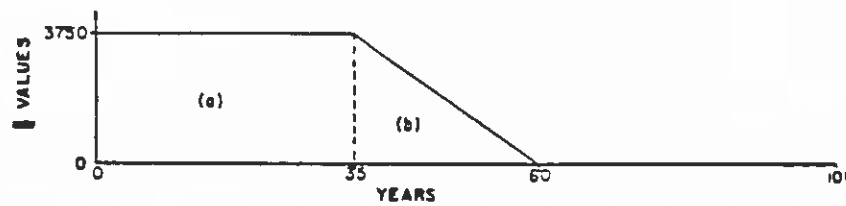
Diagram the problem:



Solution: ($500)(9.81815[1/])(.68058[2/])(.09368[3/]) = $313

Problem 4

The recreational use of a reservoir is expected to be 3,000 visits the first year of use and will remain constant for 35 years. They are then expected to decline to 0 at the 60th year of the 100-year evaluation period. Using an 8 percent interest rate, determine the average annual recreational benefits if visits are valued at $1.25 each.



Solution: a. ($3,000)($1.25) = $3,750

(3,750)(11.65457[4/]) = $43,705

b. $3,750 ÷ 25[5/] = $150 per year

(150)(179.06530[6/])(.06763[7/]) = $1,817

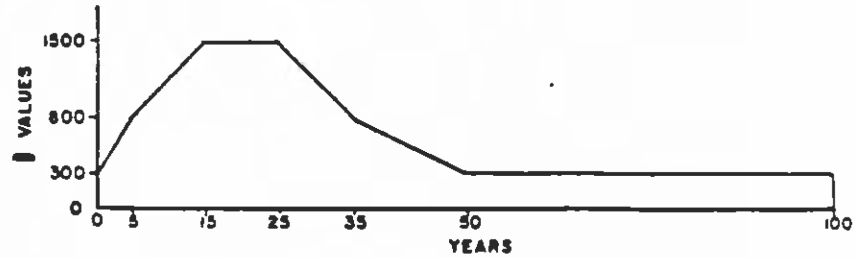
$43,705 + $1,817 = $45,522

(45,522)(.08004[8/]) = $3,644

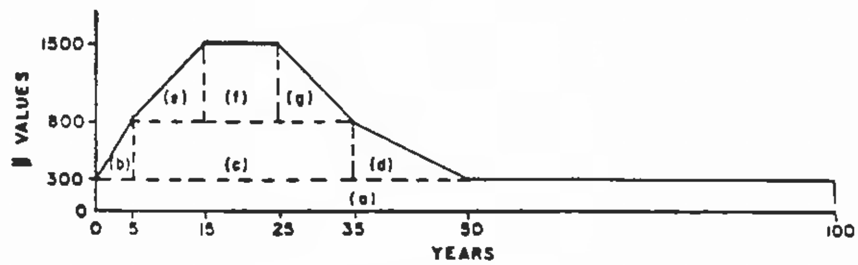
1/Present value of an annuity of 1, 20 years, 8 percent interest.
2/Present value of 1, 5 years, 8 percent interest.
3/Amortization, 25 years, 8 percent interest.
4/Present value of an annuity of 1 per year, 35 years, 8 percent interest.
5/Necessary to meet the definition of an annuity. (60 years - 35 years = 25 years.)
6/Present value of a decreasing annuity, 25 years hence, 8 percent interest.
7/Present value of 1, 35 years hence, 8 percent interest.
8/Amortization, 100 years, 8 percent interest.

Problem 5

This problem combines all of the previous principles into one example. Assume this problem involved benefits that would accrue in the following manner: begin with $300; grow to $800 in 5 years; grow to $1500 by year 15; remain constant until year 25, then decrease to $300 by year 35; decrease to $300 by year 50 and remain constant to year 100.



To solve this complicated problem, first break the problem into segments that can be easily computed.



a. Is a constant annuity[1] for the duration of the evaluation period.

b. Is a buildup period or increasing annuity for 5 years with no lag or delay.

c. Is a constant annuity[1] for the duration of the evaluation period.

d. Is a decreasing annuity for 15 years delayed 35 years.

e. Is an increasing annuity for 10 years delayed 5 years.

f. Is a constant annuity[1] for 10 years delayed 15 years.

g. Is a decreasing annuity for 10 years delayed 25 years.

Dollar values listed in the problem and the factors from the tables are multiplied together. The values for (a) thru (g) are then added together and amortized over the evaluation period to determine the average annual values. Remember where increasing and decreasing annuities are involved, you must divide the dollar value by the number of years, to get an annual rate of increase or decrease.

1/Constant annuity is referred to in the tables as "present value of an annuity of 1 per year."
2/Present value of an annuity of 1 per year, 100 years hence, 8 percent interest.
3/Necessary to meet the definition of an annuity.
4/Present value of an increasing annuity, 5 years hence, 8 percent interest.
5/Present value of an annuity of 1 per year, 30 years hence, 8 percent interest.
6/Present value of 1, 5 years hence, 8 percent interest.

Solution: The average annual value at 8 percent interest is as follows:

a.	($300)(12.49432[2/]) =	$ 3,748
b.	$800 - $300 = $500	
	$500 ÷ 5[3/] = 100	
	(100)(11.36514[4/]) =	1,137
c.	($500)(11.25778[5/])(.68058[6/]) =	3,831
d.	$500 ÷ 15[1/] = 33.33	
	(33.33)(80.50652[2/])(.06763[3/]) =	181
e.	$1,500 - $800 = $700	
	$700 ÷ 10 = $70	
	(70)(32.68691[4/])(.68058[5/]) =	1,557
f.	($700)(6.71008[6/])(.31524[7/]) =	1,481
g.	$700 ÷ 10[1/] = $70	
	(70)(41.12398[8/])(.14602[9/]) =	420

($3,748 + $1,137 + $3,831 + $181 + $1,557 + $1,481 + $420 = <u>$12,355</u>

($12,355)(.08004[10/]) = $989 Average Annual Value

1/Necessary to meet the definition of an annuity.
2/Present value of a decreasing annuity, 15 years hence, 8 percent interest.
3/Present value of 1, 35 years hence, 8 percent interest.
4/Present value of an increasing annuity, 10 years hence, 8 percent interest.
5/Present value of 1, 5 years hence, 8 percent interest.
6/Present value of an annuity of 1 per year, 10 years hence, 8 percent interest.
7/Present value of 1, 15 years hence, 8 percent interest.
8/Present value of a decreasing annuity, 10 years hence, 8 percent interest.
9/Present value of 1, 25 years, 8 percent interest.
10/Amortization, 100 years, 8 percent interest.

TABLE A. LAG IN ACCURAL FOR EVALUATION PERIOD

Evaluation Period - 50 Years

Rate of Interest	Years 5	10	15	20	25	30
3 1/8	.925	.840	.762	.692	.629	.571
3 1/4	.924	.837	.758	.687	.623	.565
4 5/8	.906	.802	.713	.635	.568	.509
4 7/8	.902	.796	.705	.626	.558	.499
5 1/8	.899	.790	.696	.617	.548	.489
5 3/8	.895	.783	.688	.608	.539	.480
5 5/8	.892	.777	.680	.599	.529	.470
5 7/8	.899	.771	.672	.590	.520	.461
6 1/8	.885	.764	.664	.581	.511	.452
6 3/8	.882	.758	.657	.572	.502	.444
6 5/8	.878	.752	.649	.564	.494	.435
6 7/8	.875	.746	.641	.556	.485	.427
7 1/8	.871	.740	.634	.547	.471	.419
7 3/8	.868	.734	.626	.539	.469	.411
7 5/8	.865	.728	.619	.532	.461	.403
7 7/8	.861	.722	.612	.524	.453	.396
8	.860	.719	.608	.520	.450	.392
8 1/8	.858	.716	.604	.516	.446	.389
8 3/8	.854	.710	.597	.509	.438	.382
8 5/8	.851	.704	.590	.501	.431	.375
8 7/8	.848	.698	.583	.494	.424	.368
9 1/8	.844	.693	.577	.487	.417	.361
9 3/8	.841	.687	.570	.480	.410	.355
9 5/8	.838	.681	.564	.474	.404	.349
9 7/8	.834	.676	.557	.467	.397	.343
10 1/8	.831	.670	.551	.460	.391	.337
10 3/8	.828	.665	.545	.454	.385	.331
10 5/8	.824	.660	.539	.448	.379	.326
10 7/8	.821	.654	.533	.442	.373	.321
11 1/8	.818	.649	.527	.436	.368	.31[illegible]
11 3/8	.815	.644	.521	.430	.362	.3[illegible]

Evaluation Period - 100 Years

Rate of Interest	5	10	15	20	25	30
3 1/8	.938	.868	.804	.747	.694	.647
3 1/4	.936	.864	.799	.740	.687	.639
4 5/8	.914	.821	.740	.670	.608	.555
4 7/8	.910	.813	.730	.658	.595	.541
5 1/8	.907	.806	.719	.646	.582	.528
5 3/8	.903	.798	.709	.634	.570	.515
5 5/8	.899	.791	.700	.623	.558	.503
5 7/8	.895	.783	.690	.612	.546	.491
6 1/8	.891	.776	.681	.601	.535	.479
6 3/8	.887	.769	.672	.591	.524	.468
6 5/8	.883	.762	.662	.581	.513	.457
6 7/8	.879	.755	.654	.571	.503	.447
7 1/8	.875	.748	.645	.562	.493	.437
7 3/8	.872	.741	.637	.552	.484	.427
7 5/8	.868	.734	.628	.543	.474	.418
7 7/8	.864	.728	.620	.534	.465	.409
8 1/8	.861	.721	.612	.526	.457	.401
8 3/8	.857	.715	.604	.517	.448	.393
8 5/8	.853	.709	.597	.509	.440	.385
8 7/8	.850	.703	.589	.501	.432	.377
9 1/8	.846	.696	.582	.494	.424	.369
9 3/8	.843	.690	.575	.486	.417	.362
9 5/8	.839	.685	.568	.479	.410	.355
9 7/8	.836	.679	.561	.472	.403	.349
10 1/8	.832	.673	.554	.465	.396	.342
10 3/8	.829	.667	.548	.458	.389	.336
10 5/8	.825	.662	.541	.451	.383	.330
10 7/8	.822	.656	.535	.445	.377	.324
11 1/8	.819	.651	.529	.439	.371	.3[illegible]
11 3/8	.816	.646	.523	.433	.365	.31[illegible]

COMPOUND INTEREST AND ANNUITY TABLES FOR
8.0000 PERCENT

NO. OF YRS. HENCE	PRESENT VALUE OF 1	AMORTI-ZATION	PRESENT VALUE OF AN ANNUITY OF 1 PER YEAR	AMOUNT OF AN ANNUITY OF 1 PER YEAR	PRESENT VALUE OF AN INCREASING ANNUITY	PRESENT VALUE OF A DECREASING ANNUITY
1	.92593	1.08000	.92593	1.00000	.92593	.92593
2	.85734	.56077	1.78326	2.08000	2.64060	2.70919
3	.79383	.3[illegible]803	2.57710	3.24640	5.02210	5.28629
4	.73503	.30192	3.31213	4.50611	7.96222	8.59841
5	.68058	.25046	3.99271	5.86660	11.36514	12.59112
6	.63017	.21632	4.62288	7.33593	15.14615	17.21400
7	.58349	.19207	5.20637	8.92280	19.23059	22.42037
8	.54027	.17401	5.74664	10.63663	23.55274	28.16701
9	.50025	.16008	6.24689	12.48756	28.05498	34.41390
10	.46319	.14903	6.71008	14.48656	32.68691	41.12398
11	.42888	.14008	7.13896	16.64549	37.40462	48.26295
12	.39711	.13270	7.53608	18.97713	42.16999	55.79902
13	.36770	.12652	7.90378	21.49530	46.95006	63.70280
14	.34046	.12130	8.24424	24.21492	51.71652	71.94704
15	.31524	.11683	8.55948	27.15211	56.44514	80.50652
16	.29189	.11298	8.85137	30.32428	61.11539	89.35789
17	.27027	.10963	9.12164	33.75023	65.70996	98.47952
18	.25025	.10670	9.37189	37.45024	70.21444	107.85141
19	.23171	.10413	9.60360	41.44626	74.61697	117.45501
20	.21455	.1[illegible]185	9.81815	45.76196	78.90794	127.27316
21	.19866	.09983	10.01680	50.42292	83.07971	137.28996
22	.18394	.09803	10.20074	55.45676	87.12640	147.49070
23	.17032	.0[illegible]642	10.37106	60.89330	91.04365	157.86176
24	.15770	.09498	10.52876	66.76476	94.82844	168.39052
25	.14602	.09368	10.67478	73.10594	98.47888	179.06530
26	.13520	.09251	10.80998	79.95442	101.99413	189.87528
27	.12519	.09145	10.93516	87.35077	105.37417	200.81044
28	.11591	.09049	11.05108	95.33883	108.61976	211.86152
29	.10733	.08962	11.15841	103.96594	111.73226	223.01992
30	.09938	.08883	11.25778	113.28321	114.71358	234.27771
31	.09202	.08811	11.34980	123.34587	117.56607	245.62751
32	.08520	.08745	11.43500	134.21354	120.29247	257.06251
33	.07889	.08685	11.51389	145.95062	122.89581	268.57640
34	.07305	.08630	11.58693	158.62667	125.37935	280.16333
35	.06763	.08580	11.65457	172.31680	127.74656	291.81790
36	.06262	.08534	11.71719	187.10215	130.00104	303.53509
37	.05799	.08492	11.77518	203.07032	132.14651	315.31027
38	.05369	.08454	11.82887	220.31595	134.18675	327.13914
39	.04971	.08419	11.87858	238.94122	136.12558	339.01772
40	.04603	.08386	11.92461	259.05652	137.96681	350.94233
41	.04262	.08356	11.96723	280.78104	139.71428	362.90957
42	.03946	.08329	12.00670	304.24352	141.37178	374.91627
43	.03654	.08303	12.04324	329.58301	142.94303	386.95951
44	.03383	.08280	12.07707	356.94965	144.43173	399.03658
45	.03133	.08259	12.10840	386.50562	145.84149	411.14498
46	.02901	.08239	12.13741	418.42607	147.17582	423.28239
47	.02686	.08221	12.16427	452.90015	148.43818	435.44666
48	.02487	.08204	12.18914	490.13216	149.63189	447.63579
49	.02303	.08189	12.21216	530.34274	150.76021	459.84796
50	.02132	.08174	12.23348	573.77016	151.82627	472.08144

8.0000 PERCENT CONTINUED

NO. OF YRS. HENCE	PRESENT VALUE OF 1	AMORTI-ZATION	PRESENT VALUE OF AN ANNUITY OF 1 PER YEAR	AMOUNT OF AN ANNUITY OF 1 PER YEAR	PRESENT VALUE OF AN INCREASING ANNUITY	PRESENT VALUE OF A DECREASING ANNUITY
51	.01974	.08161	12.25323	620.67177	152.83311	484.33467
52	.01828	.08149	12.27151	671.32551	153.78365	496.60617
53	.01693	.08138	12.28843	726.03155	154.68070	508.89461
54	.01567	.08127	12.30410	785.11408	155.52697	521.19871
55	.01451	.08118	12.31861	848.92320	156.32507	533.51732
56	.01344	.08109	12.33205	917.83706	157.07748	545.84937
57	.01244	.08101	12.34449	992.26402	157.78660	558.19386
58	.01152	.08093	12.35601	1072.64514	158.45472	570.54987
59	.01067	.08086	12.36668	1159.45676	159.08401	582.91655
60	.00988	.08080	12.37655	1253.21330	159.67656	595.29310
61	.00914	.08074	12.38570	1354.47036	160.23436	607.67880
62	.00847	.08068	12.39416	1463.82799	160.75931	620.07296
63	.00784	.08063	12.40200	1581.93423	161.25322	632.47496
64	.00726	.08058	12.40926	1709.48897	161.71780	644.88423
65	.00672	.08054	12.41598	1847.24808	162.15468	657.30021
66	.00622	.08050	12.42221	1996.02793	162.56543	669.72242
67	.00576	.08046	12.42797	2156.71016	162.95152	682.15039
68	.00534	.08043	12.43330	2330.24698	163.31434	694.58369
69	.00494	.08040	12.43825	2517.66673	163.65523	707.02194
70	.00457	.08037	12.44282	2720.08007	163.97544	719.46475
71	.00424	.08034	12.44706	2938.68648	164.27616	731.91181
72	.00392	.08031	12.45098	3174.78140	164.55854	744.36279
73	.00363	.08029	12.45461	3429.76391	164.82362	756.81740
74	.00336	.08027	12.45797	3705.14502	165.07244	769.27537
75	.00311	.08025	12.46108	4002.55662	165.30593	781.73645
76	.00288	.08023	12.46397	4323.76115	165.52502	794.20042
77	.00267	.08021	12.46664	4670.66205	165.73054	806.66705
78	.00247	.08020	12.46911	5045.31501	165.92331	819.13616
79	.00229	.08018	12.47140	5449.94021	166.10409	831.60756
80	.00212	.08017	12.47351	5886.93543	166.27360	844.08107
81	.00196	.08016	12.47548	6358.89026	166.43251	856.55655
82	.00182	.08015	12.47729	6868.60148	166.58147	869.03384
83	.00168	.08013	12.47897	7419.08960	166.72108	881.51281
84	.00156	.08012	12.48053	8013.61677	166.85190	893.99335
85	.00144	.08012	12.48197	8655.70611	166.97447	906.47532
86	.00134	.08011	12.48331	9349.16260	167.08930	918.95863
87	.00124	.08010	12.48455	10098.09561	167.19686	931.44318
88	.00114	.08009	12.48569	10906.94326	167.29760	943.92887
89	.00106	.08008	12.48675	11780.49872	167.39194	956.41562
90	.00098	.08008	12.48773	12723.93862	167.48027	968.90335
91	.00091	.08007	12.48864	13742.85371	167.56296	981.39199
92	.00084	.08007	12.48948	14843.28200	167.64037	993.88147
93	.00078	.08006	12.49026	16031.74456	167.71283	1006.37173
94	.00072	.08006	12.49098	17315.28413	167.78064	1018.86272
95	.00067	.08005	12.49165	18701.50686	167.84409	1031.35437
96	.00062	.08005	12.49227	20198.62741	167.90347	1043.84664
97	.00057	.08005	12.49284	21815.51760	167.95902	1056.33948
98	.00053	.08004	12.49337	23561.75901	168.01098	1068.83285
99	.00049	.08004	12.49386	25447.69973	168.05958	1081.32671
100	.00045	.08004	12.49432	27484.51570	168.10504	1093.82103
PERPETUITY		.08000	12.50000		168.75000	